Noise

Analog ICs

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Contents

- Chap1: Time and Frequency Domain Analysis of Noise
- Chap2: Noise in Devices & Noise in Circuits

Chap2-Outline1 : Noise in Devices

- Resistor Thermal Noise:
  - Ex: Resistor divider and RC circuits
- MOSFET Channel Thermal Noise
- MOSFET Flicker Noise

Thermal (Johnson) Noise of a Resistor

Resistor: thermal agitation → Random motion of electrons

→ Noise voltage with PSD: $\overline{v_n^2}(f) = 4kTR$ **[V²/Hz]**

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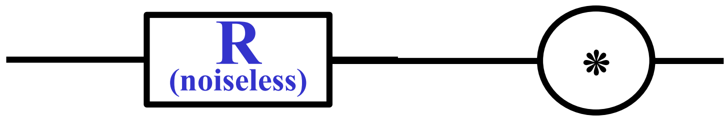
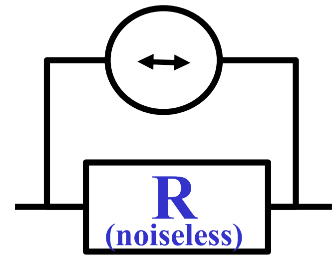


Norton representation

$\overline{i_n^2}(f) = 4kT/R$ **[A²/Hz]**



Where

k is Boltzmann constant = 1,38 10^{-23} [j/K]

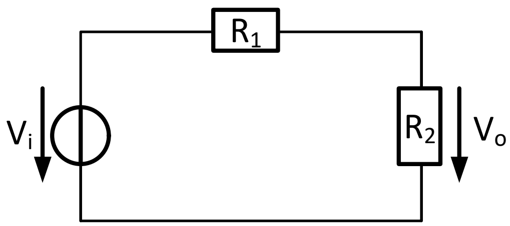
T = Temperature in [°K]

R = resistance in Ω

- Particular case: 1kΩ resistor @ 300 °K exhibits an RMS noise voltage of $\sqrt{\overline{v_n^2}} = \sqrt{4kTR} = \mathbf{4\ nV}/\sqrt{Hz}$.
- → *R = x kΩ results in a noise of* $\sqrt{\overline{v_n^2}} = \sqrt{x}\ .\mathbf{4.\ nV}/\sqrt{Hz}$

Example 3 : Resistive divider

- Determine the output noise of a resistive divider (R_1, R_2) and deduce the impact of each resistor on noise performance.
- Revaluate the noise performance using the signal-to-noise ratio (SNR).



$$\overline{v_{n,o}{}^2}(f) = 4kT(R_1//R_2)\ [V^2/Hz]$$
$$\overline{v_{n,o}{}^2} = 4kT(R_1//R_2)\ \Delta f\ [V^2]$$

Output Noise ↘ if R_1 ↘ and R_2 ↘

$$|A_v| = \frac{R_2}{R_2+R_1}$$

$$SNR = \frac{\overline{v_o{}^2}}{\overline{v_{n,o}{}^2}} = \frac{\left(\frac{R_2}{R_2+R_1}\right)^2 \overline{v_i{}^2}}{4kT\frac{R_2R_1}{R_2+R_1}\Delta f} = \frac{1}{\Delta f}\frac{R_2}{4kTR_1(R_2+R_1)}\overline{v_i{}^2} = \frac{1}{\Delta f}\frac{\overline{v_i{}^2}}{4kT}\frac{1}{R_1\left(1+\frac{R_1}{R_2}\right)}$$

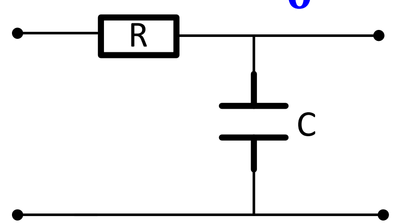
SNR ↗ if R_1 ↘ and R_2 ↗

Example 4:

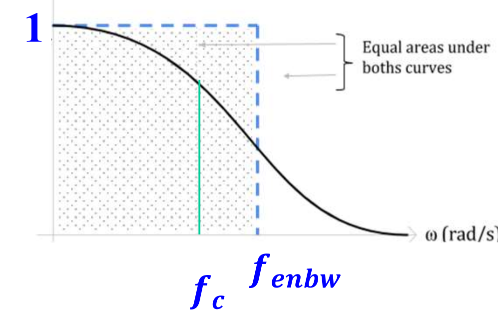
- Demonstrate that the total noise power of a simple low-pass RC circuit is: $\boxed{\overline{v_n^{\,2}} = \dfrac{kT}{C}}$
- Explain intuitively why this noise is independent of the value of R:

[V²/Hz]

$4kTR$



$$\int_0^\infty 4\mathrm{kTR}.\,|H(f)|^2 df$$



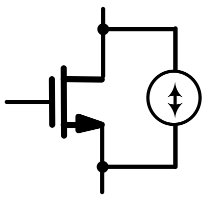
$$\overline{v_{n,o}^{\,2}} = 4kTRf_{enbw} = 4kTR\frac{\pi}{2}f_c = 4kTR\frac{\pi}{2}\frac{1}{2\pi RC} = \frac{kT}{C}$$

If R ↗ → Noise PSD ↗ but f_c ↘

Chap2-Outline1

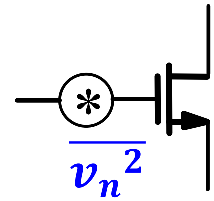
- Resistor Thermal Noise
- MOSFET Channel Thermal Noise
- MOSFET Flicker Noise

Channel Thermal Noise



$$\overline{i_n^{\,2}} = 4kT\gamma g_m \ [\mathrm{A^2/\,Hz}]$$

(in saturation)



$$\overline{v_n^{\,2}} = 4kT\gamma \frac{1}{g_m}$$

Input Voltage representation

- With $\boldsymbol{\gamma} \approx 2/3$ for long channel MOSFETs ($\rightarrow$1 for modern technologies)
- nMOS has a larger g_m and so larger channel current thermal noise
- Should we minimize or maximize g_m?

Chap2-Outline1

- Resistor Thermal Noise
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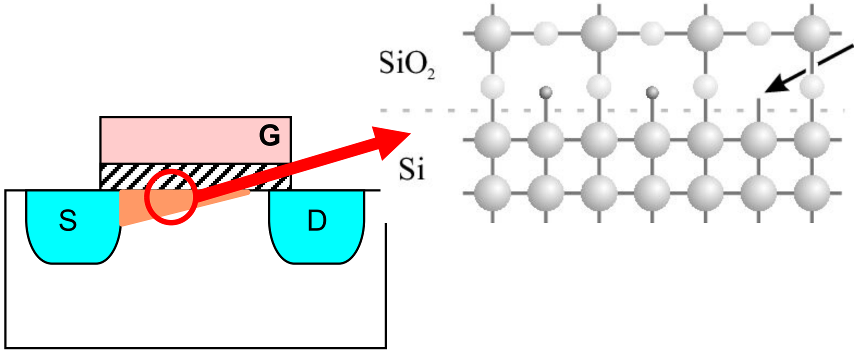
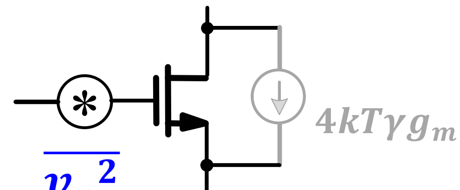
Flicker Noise

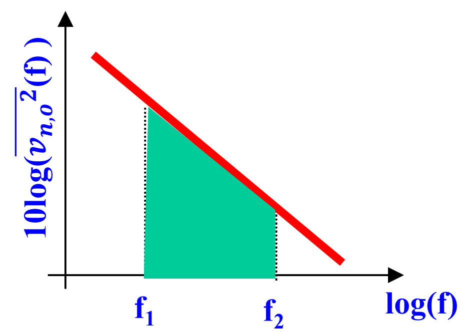


Dangling Bonds
→ Trapping and de-trapping mobile carriers
→ Flicker noise



$$\overline{v_n^2} = \frac{K}{C_{ox}LW}\cdot\frac{1}{f}$$

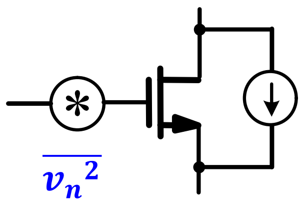
- K is a constant empirically determined
- Big transistors have less flicker noise
- K is generally lower for PMOS



- Ex: The total noise power from f_1 to f_2 [V^2]:

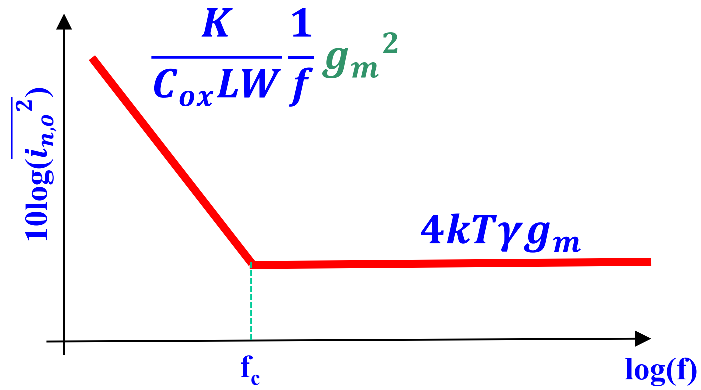
$$\overline{v_n^2} = \int_{f_1}^{f_2} \frac{K}{C_{ox}LW}\cdot\frac{1}{f}df = \frac{K}{C_{ox}LW}\cdot Ln\left(\frac{f_2}{f_1}\right)$$

Total Noise in MOSFETs



$\overline{i_n^2} = 4kT\gamma g_m$

$\overline{v_n^2} = \frac{K}{C_{ox}LW}\cdot\frac{1}{f}$

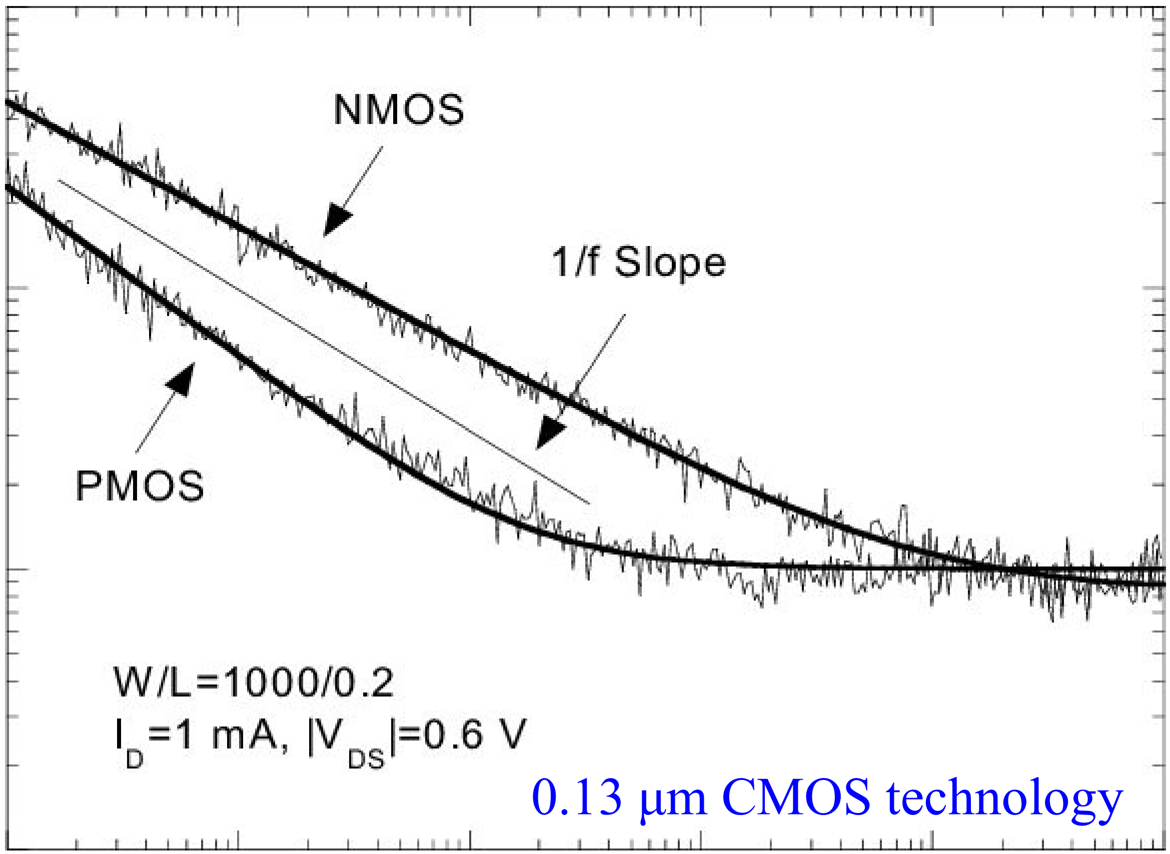


f_c: Flicker noise "corner frequency": usually ϵ [500 kHz, 1 MHz]

$$f_c = \frac{K}{c_{ox}WL}g_m\frac{3}{8kT}$$

- Flicker noise is detrimental for low-frequency circuits
- Or if it is upconverted the signal Band.

nMOS vs pMOS

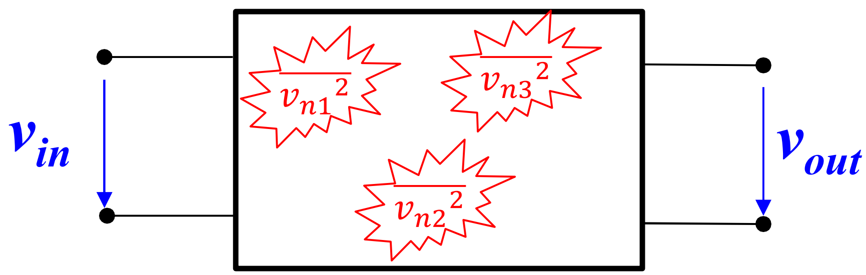


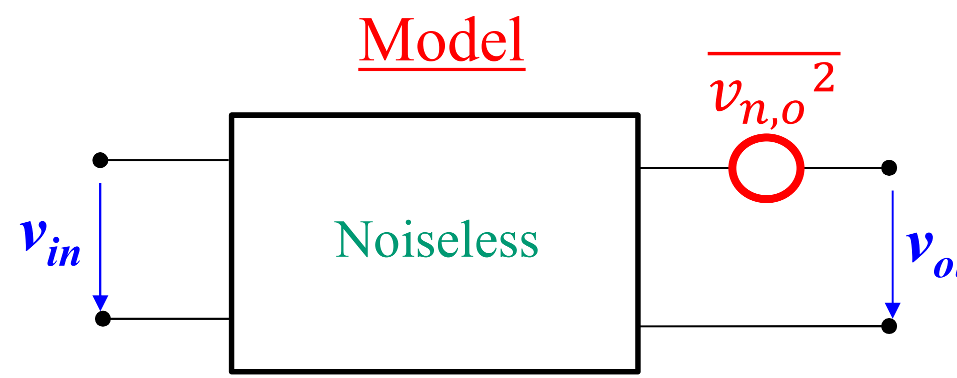
NMOS
1/f Slope
PMOS
W/L=1000/0.2
I_D=1 mA, $|V_{DS}|$=0.6 V
0.13 μm CMOS technology

Chap2-Outline2: Noise in Circuits

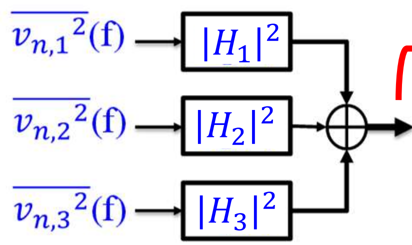
- Circuit-Noise Analysis and Modeling
  - Example: Noise in CS Amplifier
- Input-Referred Noise
  - Effect of source impedance
- Example: Noise CS, CG amplifier and source follower.

Circuit-Noise Analysis and Modeling Procedure





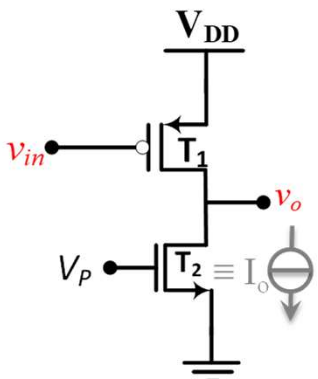
- Identify noise sources.
- Determine ($H_i(s)$) from $\overline{v_{ni}^2}$ to the output.
- Add all of the contributions to the output



$$\overline{v_{n,o}^2}(\mathrm{f}) = \sum_{j=1}^{3} \overline{v_{n,j}^2}(\mathrm{f})\,|H_j(f)|^2$$

Example: Noise in CS Amplifier

- Determine the output thermal noise voltage of a common source amplifier and deduce the impact of g_{m1} and g_{m2} on noise performance.



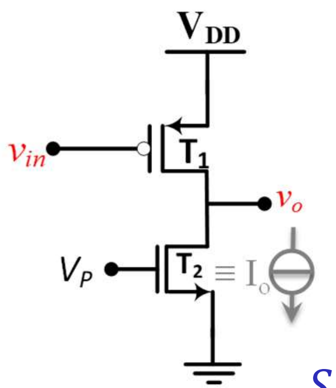
$$\overline{i_{n1,2}}^{2} = 4kT\gamma g_{m1,2} \text{ [A}^2\text{/ Hz]}$$

$$\overline{v_{n,o}}^{2} = 4kT\gamma(g_{m1} + g_{m1})(r_{o1}//r_{o2})^2 \text{ [V}^2\text{/ Hz]}$$

Output Noise ↘ if g_{m1} ↘ and g_{m2} ↘

Example: Noise in CS Amplifier

- Revaluate the noise performance using the signal-to-noise ratio (SNR).



$$|A_v| = g_{m1}\, r_{o1} // r_{o2}$$

$$\overline{v_{n,o}^{\ 2}} = 4kT\gamma(g_{m1} + g_{m2})(r_{o1} // r_{o2})^2 \ [\mathrm{V^2/\,Hz}]$$

$$SNR = \frac{\overline{v_o^{\ 2}}}{\overline{v_{n,o}^{\ 2}}} = \frac{|A_v|^2\overline{v_i^{\ 2}}}{\overline{v_{n,o}^{\ 2}}} = \frac{g_{m1}^{\ 2}}{4kT\gamma(g_{m1}+g_{m2})\Delta f}\overline{v_i^{\ 2}} = \frac{1}{\Delta f}\frac{g_{m1}}{4kT\gamma\left(1+\frac{g_{m2}}{g_{m1}}\right)}\overline{v_i^{\ 2}}$$

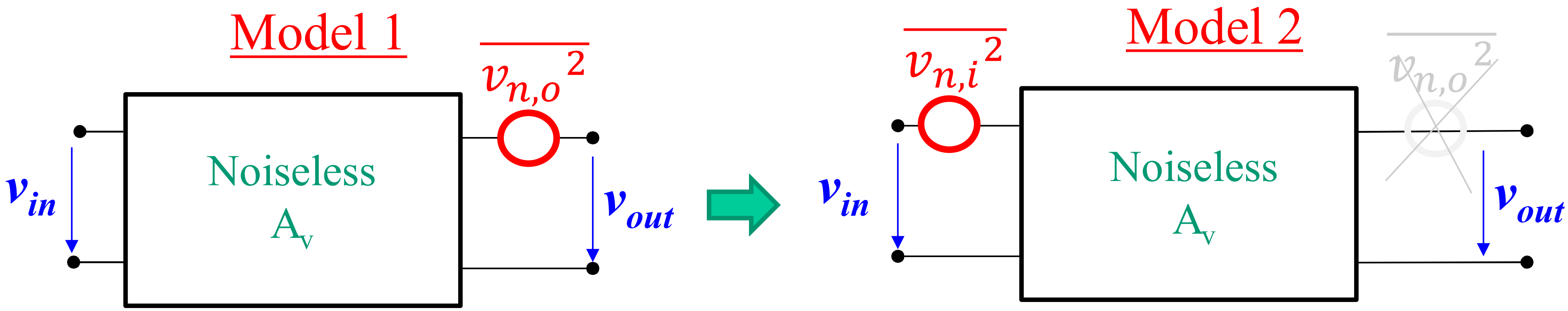
$SNR \nearrow$ if $g_{m1} \nearrow$ and $g_{m2} \searrow$

Conclusion: improve noise performances → maximize g_m of active transistor and minimize g_m of load (current source)

Chap2-Outline2

- Circuit-Noise Analysis and Modeling
  – Example: Noise in CS Amplifier
- Input-Referred Noise
  – Effect of source impedance
- Example: Noise CS, CG amplifier and source follower.

Input-Referred Noise



A_v is the voltage gain

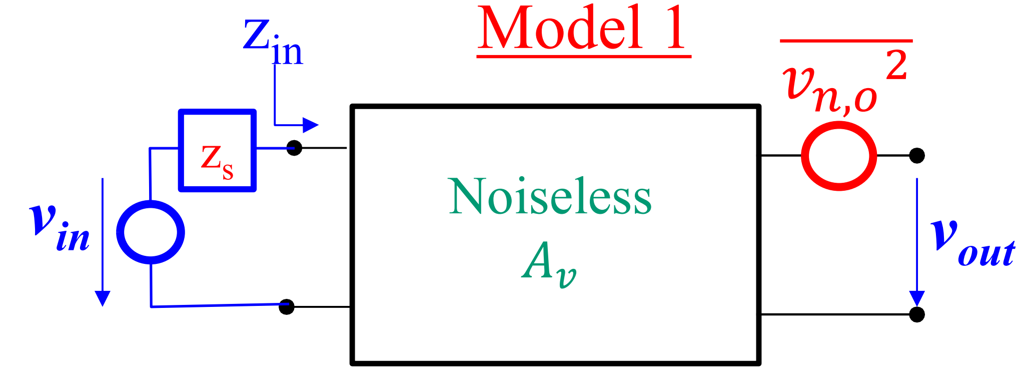
$$\boxed{\overline{v_{n,i}^2} = \frac{\overline{v_{n,o}^2}}{A_v^2}}$$

- Why is $\overline{v_{n,i}^2}$ important?:
  - Optimizing $\overline{v_{n,o}^2}$ can degrade the gain (A_v) and so the desired signal

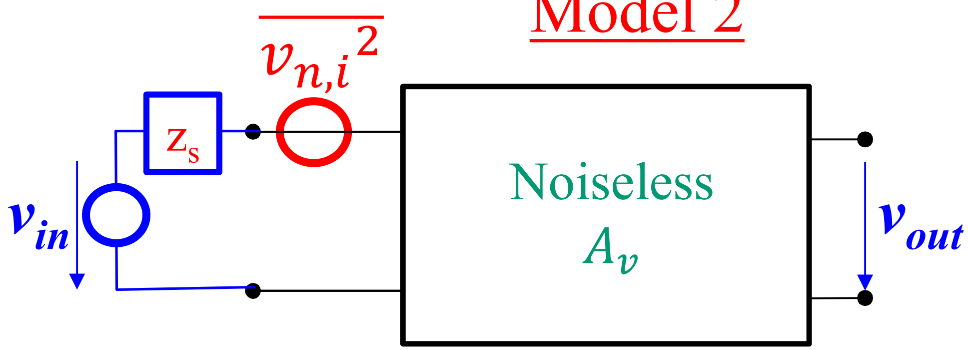
    → Better optimize $\overline{v_{n,i}^2}$

Effect of source Impedance on Input-Referred Noise

Model 1



Model 2



$$\overline{v_{n,i}{}^2} = \frac{\overline{v_{n,o}{}^2}}{\left(A_v \frac{z_{in}}{z_{in}+z_s}\right)^2}$$

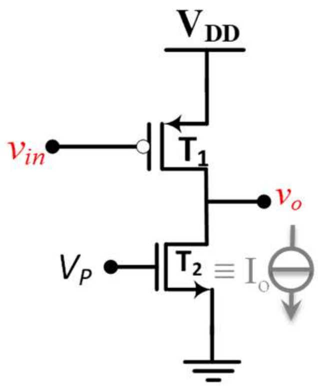
Note:

z_s have no effect if $z_{in} \rightarrow \infty$

(e.g. CS and CD at low-frequency)

Example: Noise in CS Amplifier

- Determine the input-referred voltage noise of CS amp at low frequency (flicker and thermal).



$$|A_v| = g_{m1}\, r_{o1}//r_{o2}$$

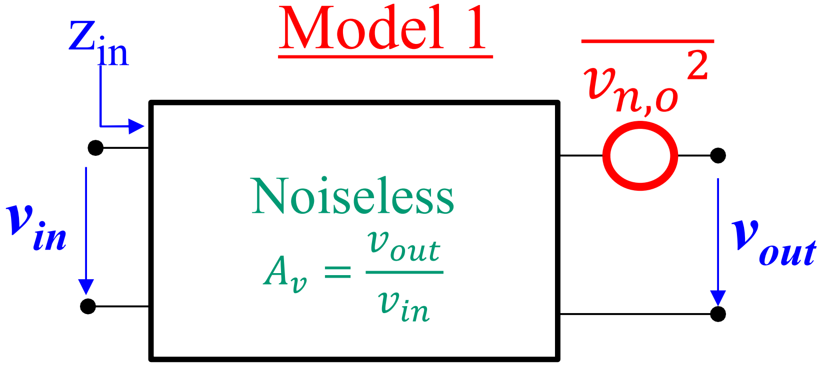
$$\overline{v_{n,o}{}^2} = 4kT\gamma(g_{m1} + g_{m2})(r_{o1}//r_{o2})^2 + |A_v|^2 \frac{K}{C_{ox}(LW)_1}\cdot\frac{1}{f} + \left(\frac{g_{m2}}{g_{m1}}\right)^2 |A_v|^2 \frac{K}{C_{ox}(LW)_2}\cdot\frac{1}{f}$$

$$\overline{v_{n,i}{}^2} = \frac{\overline{v_{n,o}{}^2}}{|A_v|^2} = 4kT\gamma\frac{1}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}}\right) + \frac{K}{C_{ox}(LW)_1}\cdot\frac{1}{f} + \left(\frac{g_{m2}}{g_{m1}}\right)^2 \frac{K}{C_{ox}(LW)_2}\cdot\frac{1}{f}\ \ [V^2/\,Hz]$$

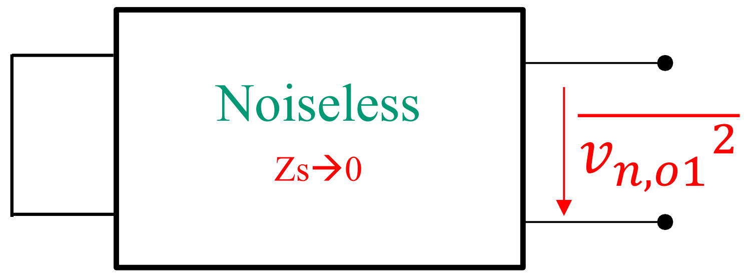
Conclusion: improve noise performances → maximize g_m of active transistor and minimize g_m of load (current source)

Complete Input-Referred Noise Model : Valid ∀ z_s

Model 1

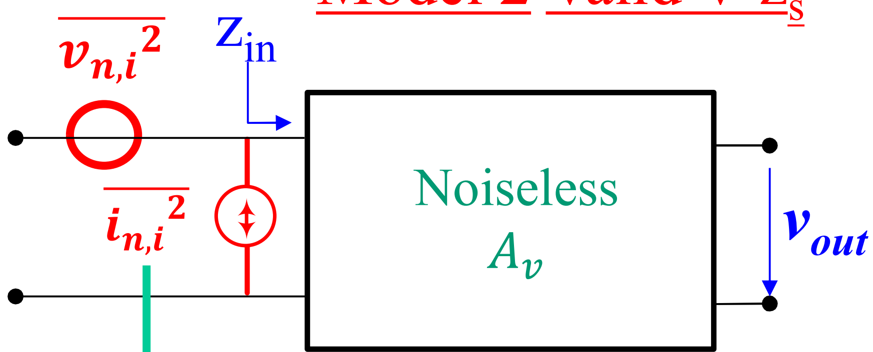


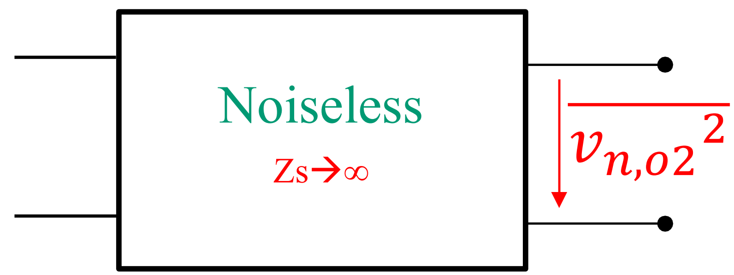
A_v is the voltage gain



$$\overline{v_{n,i}^2} = \frac{\overline{v_{n,o1}^2}}{(A_v)^2}$$

Model 2 Valid ∀ z_s



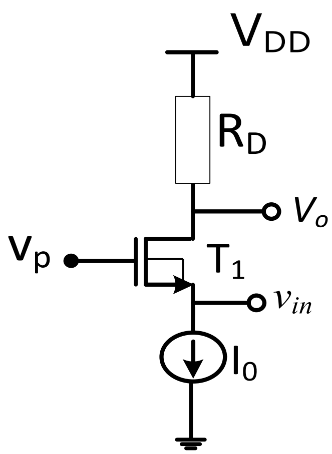


$$\overline{i_{n,i}^2} = \frac{\overline{v_{n,o2}^2}}{(z_{in} A_v)^2}$$

Correlated

Example: Noise in CG Amplifier

- Determine the output and input-referred thermal noise voltage and current of a CG amp (neglect the noise of I_0).



$$|A_v| = g_{m1}\, r_{o1} // R_D \approx g_{m1} R_D$$

$$Z_{in} = \frac{1}{g_{m1}}\left(1 + \frac{R_D}{r_{o1}}\right) // r_{o2} \approx \frac{1}{g_{m1}}$$

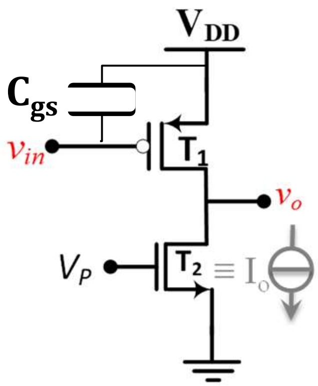
$$\overline{v_{n,0}{}^2} = 4kT\gamma g_{m1} R_D{}^2 + 4kTR_D$$

$$\overline{v_{n,i}{}^2} = \frac{\overline{v_{n,o1}{}^2}}{|A_v|^2} = 4kT\gamma \frac{1}{g_{m1}} + 4kT \frac{1}{R_D (g_{m1})^2} \quad \textit{(short –circuit the input)}$$

$$\overline{i_{n,i}{}^2} = \frac{\overline{v_{n,o2}{}^2}}{|z_{in} A_v|^2} = 4kT \frac{1}{R_D} \; \textit{(open the input)}$$

Example: Noise in CS Amplifier

- Determine the input-referred current noise of CS amp at high frequency (consider Cgs capacitor).

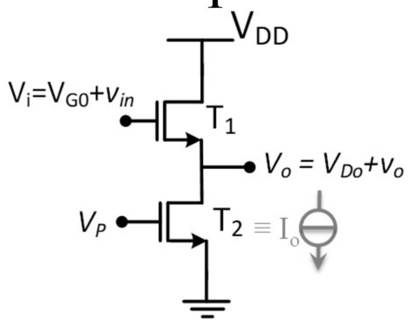


$$Z_{in} = \frac{1}{j2\pi f C_{gs}}$$

$$\overline{i_{n,i}^{\,2}} = \frac{\overline{v_{no,2}^{\,2}}}{|Z_{in}A_v|^2} = \left(2\pi f C_{gs}\right)^2 4kT\gamma \frac{1}{g_{m1}}\left(1+\frac{g_{m2}}{g_{m1}}\right)$$
$$+\left(2\pi f C_{gs}\right)^2 \left(\frac{g_{m2}}{g_{m1}}\right)^2 \frac{K}{C_{ox}(LW)_2}\cdot\frac{1}{f}\ \ [\mathrm{A^2/\,Hz}]$$

Example: Noise in CD Amplifier (Source Follower)

- Determine the output and input-referred thermal noise of a Source Follower at low frequency (flicker and thermal) and comment on its noise performances.



$$|A_v| = 1$$

$$\overline{v_{n,o}{}^2} = 4kT\gamma(g_{m1}+g_{m2})\left(\frac{1}{g_{m1}}\right)^2 + \frac{K}{C_{ox}(LW)_1}\cdot\frac{1}{f} + \frac{K}{C_{ox}(LW)_2}\cdot\frac{1}{f}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

$$\overline{v_{n,i}{}^2} = \frac{\overline{v_{n,o}{}^2}}{|A_v|^2} = 4kT\gamma\frac{1}{g_{m1}}\left(1+\frac{g_{m2}}{g_{m1}}\right) + \frac{K}{C_{ox}(LW)_1}\cdot\frac{1}{f} + \left(\frac{g_{m2}}{g_{m1}}\right)^2\frac{K}{C_{ox}(LW)_2}\cdot\frac{1}{f}\ \ [\mathrm{V^2/\,Hz}]$$

Note: $\overline{v_{n,i}{}^2}$ of CD is similar to CS but with no gain for the desired signal